MAE 606 Prelim 2 **April 5, 2005**

Unactuated and Actuated Spacecraft

1. (40 points) The rigid spacecraft shown in Figure 1 has a single, fixed, symmetric rotor whose inertia dyadic for its mass center is

$$\boldsymbol{I}_W = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_3 \end{bmatrix} \begin{bmatrix} I_T & 0 & 0 \\ 0 & I_T & 0 \\ 0 & 0 & I_S \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \end{bmatrix}.$$

The inertia dyadic $\boldsymbol{I}_B$ for the rigid body (not including the rotor) for its mass center, is

$$\boldsymbol{I}_B = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_3 \end{bmatrix} \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_1 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \end{bmatrix}.$$

The mass centers of the two bodies coincide. The rotor is constrained (with bearings) such that it can rotate only about $\boldsymbol{b}_3$. A viscous torque $\boldsymbol{\tau}_d$ acts between the rotor and the spacecraft:

$$\boldsymbol{\tau}_d = \tau_d \boldsymbol{b}_3 \text{ (a vector),}$$

where

$$\tau_d = c\boldsymbol{b}_3 \cdot \left(\boldsymbol{\omega}^{\text{D/N}} - \boldsymbol{\omega}^{\text{B/N}}\right) \text{ (a scalar).}$$

Specifically, $\overset{\text{N}}{\dot{\boldsymbol{H}}}_B = \boldsymbol{\tau}_d$ and $\overset{\text{N}}{\dot{\boldsymbol{H}}}_W = -\boldsymbol{\tau}_d$, where $\overset{\text{N}}{\dot{\boldsymbol{H}}}_B$ is the derivative of the spacecraft body's angular momentum in an inertial frame N, and $\overset{\text{N}}{\dot{\boldsymbol{H}}}_W$ is the derivative of the rotor's angular momentum in N. Consider the case of c>0 and the initial conditions $\boldsymbol{\omega}^{\text{B/N}}(0) = 0.1\boldsymbol{b}_2 + 1\boldsymbol{b}_3$ and $\boldsymbol{\omega}^{\text{D/N}}(0) = 0.1\boldsymbol{b}_2$.

a. (15 points) Write the equations of motion. These equations should consist of a vector equation for the motion of the rigid body and either a scalar or a vector equation for the motion of the rotor.

b. (5 points) Is $\boldsymbol{b}_3 \cdot (\boldsymbol{H}_B + \boldsymbol{H}_W)$ constant?

c. (10 points) Use the results of parts a. and b. to solve explicitly for $\boldsymbol{b}_3 \cdot \overset{\text{B}}{\dot{\boldsymbol{\omega}}}^{\text{B/N}}(t)$

d. (10 points) Can this rotor damp nutation for this spacecraft?

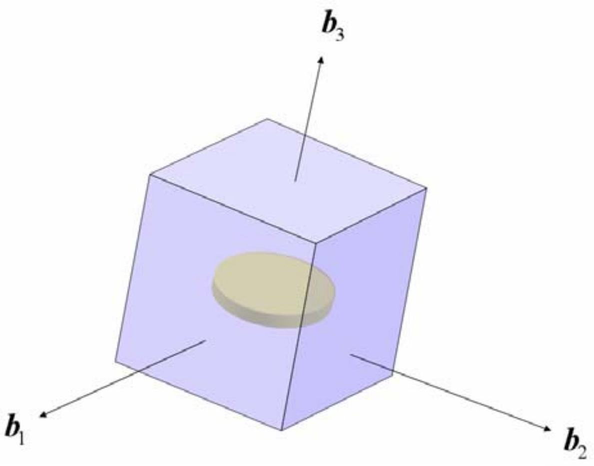

Figure 1. Spacecraft with a Fixed Rotor

2. (10 points) The gyro data shown in Figure 2 is from a rigid spacecraft with virtually no nutation damping.

a. (5 points) Given ${}^{B}I_{11}$ =350 kg-m^2, estimate ${}^{B}I_{13}$, ${}^{B}I_{23}$, ${}^{B}I_{22}$ and ${}^{B}I_{33.}$

b. (5 points) Is this spacecraft behaving as a major-axis spinner or as a minor-axis spinner? Why?

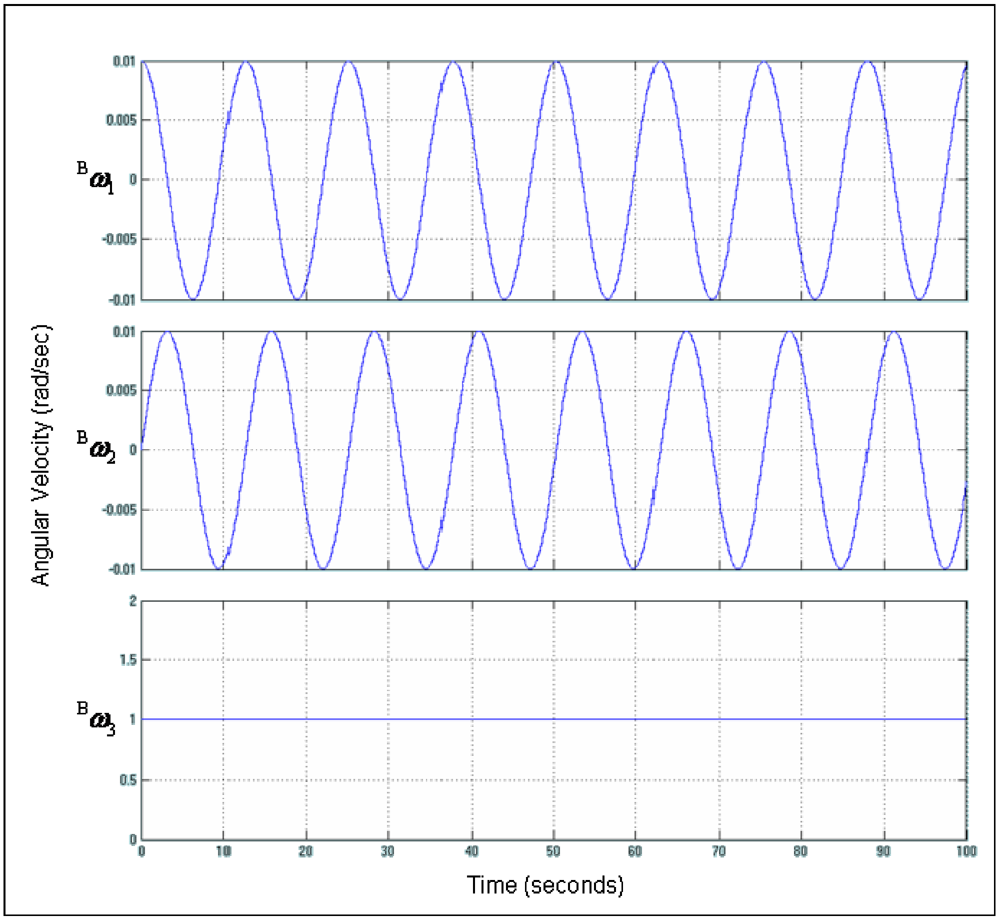

Figure 2. Gyro Rates for a Rigid Spacecraft.

3. (20 points) An array of four reaction wheels has as its Jacobian

$$A = \frac{\sqrt{2}}{2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

such that

$$^{\mathrm{B}}h = A\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix},$$

where $^{\mathrm{B}}h$ is the total wheel momentum in a spacecraft body-fixed rotating frame B in terms of some body-fixed basis vectors. The maximum angular momentum possible in a single wheel is 100 Nms.

Consider the following sets of wheel angular-momentum values:

$$h_\alpha = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} 15 \\ 30 \\ -20 \\ -25 \end{bmatrix} \text{ Nms} \quad \text{and} \quad h_\beta = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} 17.5 \\ 27.5 \\ -17.5 \\ -27.5 \end{bmatrix} \text{ Nms.}$$

a. (5 points) Does $h_\alpha - h_\beta$ lie entirely in the null space of A?

b. (10 points) Find h_2, h_3 and h_4 such that $^{\mathrm{B}}h = \begin{bmatrix} 10\sqrt{2} \\ 10\sqrt{2} \\ 10\sqrt{2} \end{bmatrix}$ Nms and h_1=100 Nms. There are several ways to solve this problem, but using the null space and a pseudoinverse may be the easiest by hand.

c. (5 points) In what situations is this array singular?

4. (15 points) Consider a rigid gyrostat with

$$\boldsymbol{I}_C = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_3 \end{bmatrix} \begin{bmatrix} 300 & 0 & 0 \\ 0 & 400 & 5 \\ 0 & 5 & 500 \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \end{bmatrix} \text{ kg-m}^2$$

and some wheel angular momentum $\boldsymbol{h}=100\boldsymbol{b}_1$. Let $\boldsymbol{\omega}^{\mathrm{B/N}} = 0.1\boldsymbol{b}_1$, and assume that the spacecraft is intended to spin about $\hat{\boldsymbol{v}} = \boldsymbol{b}_1$.

a. (5 points) What is the cone angle?

b. (3 points) What is the nutation angle?

c. (2 points) What is the wobble angle.

d. (5 points) Is $\boldsymbol{\omega}^{\mathrm{B/N}} = 0.1\boldsymbol{b}_1$ a stable relative equilibrium?

Circle the letter corresponding to the correct answer.

5. (2 points) Consider a rigid gyrostat whose array of wheels can be given an arbitrary angular-momentum vector. Which of the following objectives cannot be achieved by establishing constant wheel angular momentum?
   a. Energy dissipation
   b. Dynamic balance
   c. Stable spin about any single, chosen axis
   d. A momentum bias
   e. Dual-spin –like stability

6. (2 points) The maximum angular momentum available from a collection of fixed reaction wheels can be depicted as a surface. Which of the following is not true of this surface?
   a. It is a polyhedron.
   b. The surface varies depending on the wheel speeds.
   c. The surface lies in a plane if there are only two reaction wheels.
   d. The surface looks like a cube if there are three identical reaction wheels oriented along mutually orthogonal axes.
   e. For a finite number of wheels, it is never a sphere.

7. (2 points) Which of the following is not true of a dual-gimbal CMG (DGCMG)?
   a. At some instant a DGCMG can apply torque anywhere in a plane perpendicular to its angular-momentum vector.
   b. A DGCMG generally offers less torque per Watt than a single-gimbal CMG with an identical rotor and gimbal-motor design.
   c. The surface that describes a DGCMG's angular-momentum envelope is a sphere.
   d. A pair of DGCMGs driven with opposite gimbal rates is known as a "scissored pair."
   e. An array of two DGCMGs can provide three-axis attitude control.

8. (2 points) Which of the following combination of actuators and sensors is most likely to be used on a single spacecraft?
   a. One single-gimbal CMG, three reaction wheels, and a star tracker.
   b. Four single-gimbal CMGs, a magnetometer, and an earth sensor,
   c. Four reaction wheels, an earth sensor, and a sun sensor.
   d. A gravity-gradient boom and a star tracker.
   e. A gravity-gradient boom, two single-gimbal CMGs, and a magnetometer.

9. (2 points) Which of the following is not a consequence of the Iorillo Criterion?
   a. The "Hughes Gyrostat System" is stable.
   b. Momentum-bias spacecraft are stable.
   c. Energy-dissipation mechanisms (such as fuel tanks and nutation dampers) can be placed arbitrarily in a dual-spin spacecraft.
   d. Tony Iorillo became Senior Vice President of Hughes Aircraft Company.
   e. The rate of energy dissipation on the rotor of a dual-spin spacecraft is carefully analyzed.

10. (5 points) Which of the following are not physically realizable inertia matrices?

a. The 3×3 identity matrix

b. The matrix $\begin{bmatrix} 4000 & 0 & -1000 \\ 0 & 4000 & 0 \\ -1000 & 0 & 4000 \end{bmatrix}$

c. A matrix with three real eigenvalues, all between 5 and 7

d. The matrix $\begin{bmatrix} 1000 & 0 & 0 \\ 0 & 2000 & 0 \\ 0 & 0 & 3500 \end{bmatrix}$

e. A matrix with a complex-conjugate pair of eigenvalues