MAE 606

Homework #8

HW 7 and HW 8 will be counted as a single homework, due on 3/15/05.

Use the simulation from HW 7 to model the dynamics of a spacecraft with four reaction wheels. Assume that each wheel is perfectly symmetric about its spin axis and that its mass center lies on its spin axis. The inertia of the i^{th} wheel about its spin axis, $I_{W,i}$, is such that at 6000 RPM, the wheel's angular momentum along its spin axis is 70 Nms. Except for their orientations in B and their speeds, all wheels are identical (so $I_{W,1} = I_{W,2} = I_{W,3} = I_{W,4}$). For this model, assume that the reaction-wheel speeds are constant in B. I.e., for a frame W_i fixed in the i^{th} wheel, the angular velocity $\boldsymbol{\omega}^{W_i/B}$ is entirely along the spin axis $\hat{\boldsymbol{a}}_i$ and is given by

$$\boldsymbol{\omega}^{W_i/B} = \omega_i \hat{\boldsymbol{a}}_i .$$

The composite inertia dyadic (including the rigid body's dyadic, the Kane damper's dyadic, and the dyadic of each of the four wheels) is

$$\boldsymbol{I}_c = \boldsymbol{I}_B + \boldsymbol{I}_D + \sum_{i=1}^{4} \boldsymbol{I}_{W,i} = [\boldsymbol{b}_1 \quad \boldsymbol{b}_2 \quad \boldsymbol{b}_3] \begin{bmatrix} 3000 & 0 & -30 \\ 0 & 4000 & 20 \\ -30 & 20 & 5000 \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \end{bmatrix}.$$

The wheel spin axes are those of NASA's popular "ortho-skew" architecture:

$$\hat{\boldsymbol{a}}_1 = \boldsymbol{b}_1,\ \hat{\boldsymbol{a}}_2 = \boldsymbol{b}_2,\ \hat{\boldsymbol{a}}_3 = \boldsymbol{b}_3 \text{ and } \hat{\boldsymbol{a}}_4 = \frac{\sqrt{3}}{3}\boldsymbol{b}_1 + \frac{\sqrt{3}}{3}\boldsymbol{b}_2 + \frac{\sqrt{3}}{3}\boldsymbol{b}_3$$

The objective of this exercise is to demonstrate dynamic balance. You are to select a set of constant wheel speeds that result in a stable relative equilibrium

$$\boldsymbol{\omega}_{equil}^{B/N} = \Omega \boldsymbol{b}_3$$

where Ω is the angular rate that results from the initial conditions $\boldsymbol{\omega}^{B/N}(0) = 0.5\boldsymbol{p}_3$ rad/sec. Here, $\boldsymbol{p}_3$ refers to the principal axis of $\boldsymbol{I}_C$ closest to $\boldsymbol{b}_3$. As for the damper, let $\boldsymbol{\omega}^{B/N}(0) = \boldsymbol{\omega}^{D/N}(0)$, $\boldsymbol{I}_D = 100(\boldsymbol{b}_1\boldsymbol{b}_1 + \boldsymbol{b}_2\boldsymbol{b}_2 + \boldsymbol{b}_3\boldsymbol{b}_3)$, and c=100.

Produce the following results:

1. Compute the angular velocity of the system in equilibrium.
2. Run the model for 1000 seconds with these initial conditions and your chosen wheel speeds. Produce a plot of the components of ${}^{B}\omega^{B/N}$ that shows the spacecraft converging on the desired stable relative equilibrium.

Note that the angular momentum $\boldsymbol{h}$ of the wheels in B-fixed axes $\boldsymbol{b}_{\text{i}}$ is given by

$$\boldsymbol{h} = \sum_{i=1}^{4} \boldsymbol{I}_{W,i} \cdot \boldsymbol{\omega}^{\mathrm{W_i/B}} = \begin{bmatrix} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_3 \end{bmatrix} \begin{bmatrix} {}^{\mathrm{B}}h_1 \\ {}^{\mathrm{B}}h_2 \\ {}^{\mathrm{B}}h_3 \end{bmatrix},$$

where

$$\begin{bmatrix} {}^{\mathrm{B}}h_1 \\ {}^{\mathrm{B}}h_2 \\ {}^{\mathrm{B}}h_3 \end{bmatrix} = \begin{bmatrix} I_{W,1}\,{}^{\mathrm{B}}\hat{a}_1 & I_{W,2}\,{}^{\mathrm{B}}\hat{a}_2 & I_{W,3}\,{}^{\mathrm{B}}\hat{a}_3 & I_{W,4}\,{}^{\mathrm{B}}\hat{a}_4 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

and $I_{W,i}$ is the moment of inertia of every wheel about its spin axis. For simplicity, we can write

$$\begin{bmatrix} {}^{\mathrm{B}}h_1 \\ {}^{\mathrm{B}}h_2 \\ {}^{\mathrm{B}}h_3 \end{bmatrix} = A \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}.$$

So, if we know the wheel angular momentum as the components ${}^{\mathrm{B}}h$, we can find the wheel speeds from a pseudoinverse:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = A^{+} \begin{bmatrix} {}^{\mathrm{B}}h_1 \\ {}^{\mathrm{B}}h_2 \\ {}^{\mathrm{B}}h_3 \end{bmatrix}.$$